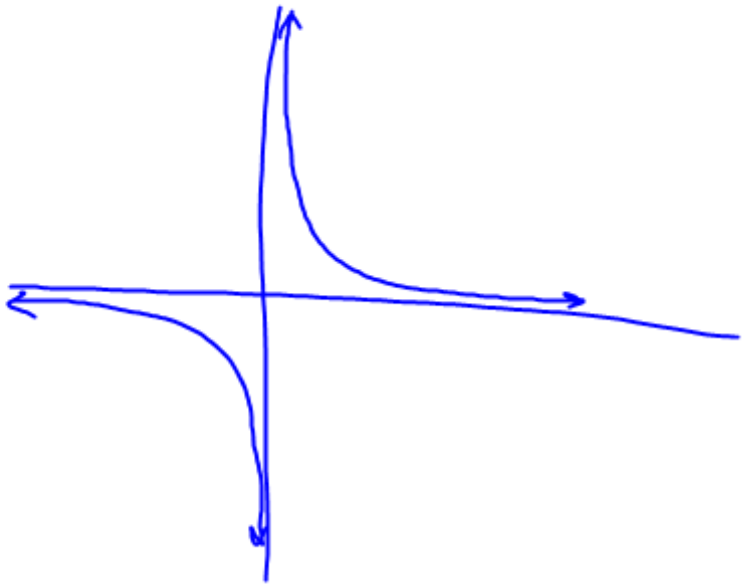
Asymptote – A line the graph approaches, but never touches – gets infinitely close

$f(x) = \frac{1}{x}$



horizontal $y = 0$

vertical $x = 0$

Rational Function

$$\frac{\text{degree } n}{\text{degree } m}\ \frac{N(x)}{D(x)} = \frac{a_1x^n + a_2x^{n-1} + \ldots + a_{n-1}x + a_n}{b_1x^n + b_2x^{n-1} + \ldots + b_{n-1}x + b_n}$$

$\frac{1}{10}$ $\frac{1}{100}$ $\frac{1}{1000000}$

Asymptotes

① Vertical asymptotes happen at the zeros of $D(x)$ $x =$

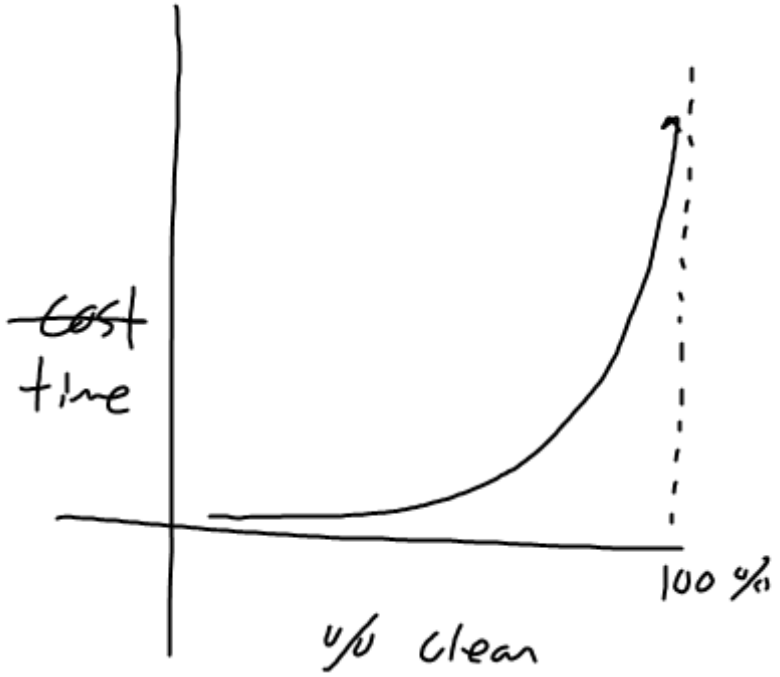
② Horizontal asymptotes – end behavior, $x \to \infty$

ⓐ if $m > n$, $y = 0$ is a horz. asymptote $\frac{1}{x}$ or $\frac{x+3}{x^2-1}$

ⓑ if $m = n$, then horz. asymptote is ratio of leading coefficients $\frac{2x+3}{3x+8}$ $y = \frac{2}{3}$

ⓒ if $m < n$, then there is no horz. asymptote $\frac{2x^2-5}{x+10}$


UV
intensity
time in sun


~~cost~~
time
100 %
% clean

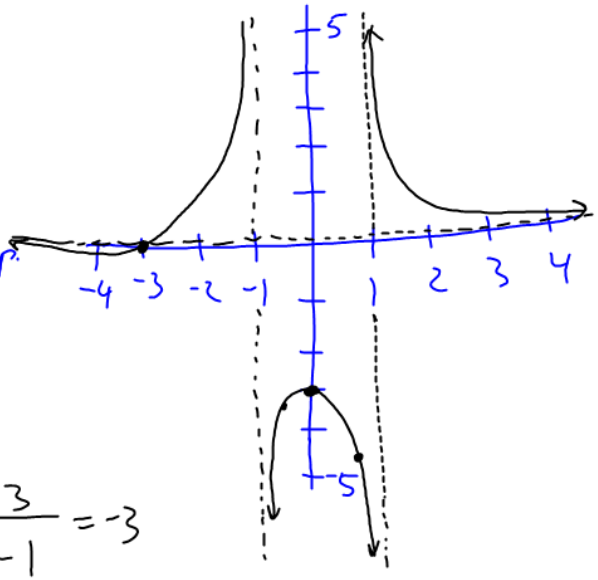
Graph $f(x) = \frac{x+3}{x^2-1}$ →  $n=1$, $m=2$

- vertical asymptote

  Zeros of $D(x)$ $x^2-1=0$, $x^2=1$, $x=\pm 1$

- Horizontal asymptote

  $Y=0$ is horz. asymp.



- y-intercept

  Set $x=0$ $\quad \frac{0+3}{0^2-1} = -3$

- x-intercepts

  Zeros of $N(x)$

  $x+3=0$

  $x=-3$

- Plot points

  $\frac{\frac{1}{2}+3}{(\frac{1}{2})^2-1} = \frac{+\frac{7}{2}}{-\frac{3}{4}} = -\frac{28}{6} = -4.\overline{66}$

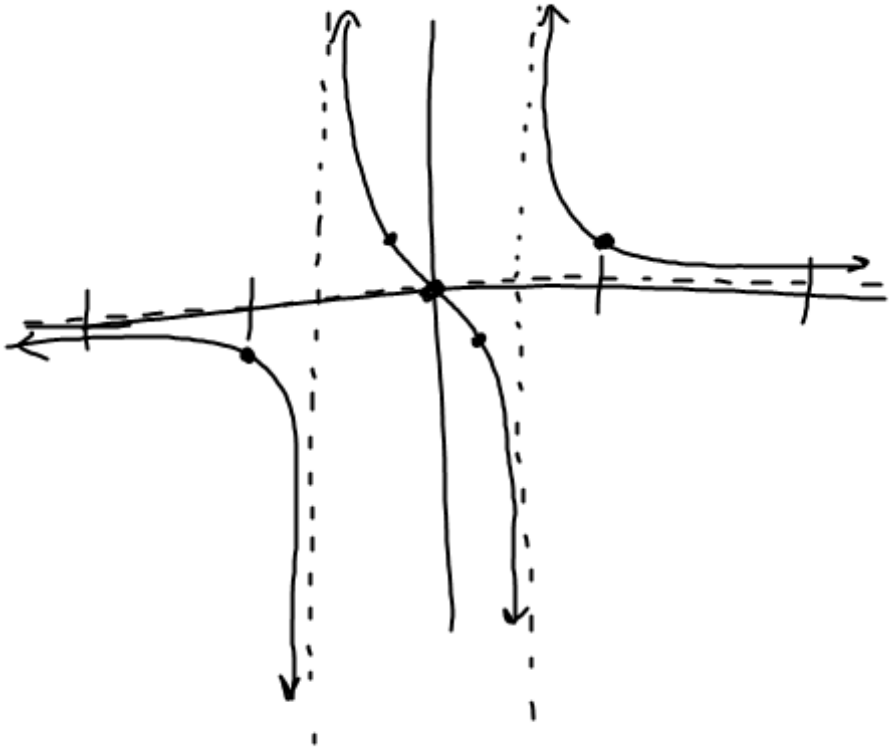
  $\frac{2+3}{2^2-1} = \frac{5}{3}$

  $-\frac{1}{2}+3 = \frac{\frac{5}{2}}{\frac{3}{4}} = -\frac{20}{6} = 3.\overline{33}$

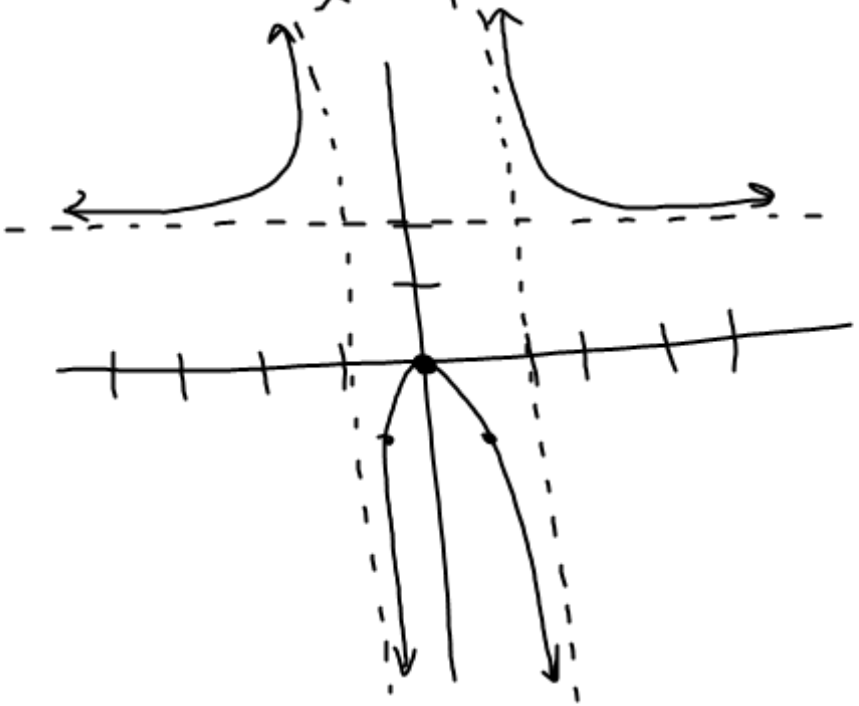
  $\frac{-2+3}{(-2)^2-1} = \frac{1}{3}$

Graph

(a) $f(x) = \dfrac{2x}{3x^2-1}$



(b) $f(x) = \dfrac{2x^2}{x^2-1}$



Sect. 2.6

#1-6 (b & c)

#7-12